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**FITTING AERODYNAMIC FORCES IN THE
LAPLACE DOMAIN: AN APPLICATION OF A
NONLINEAR NONGRADIENT TECHNIQUE TO
MULTILEVEL CONSTRAINED OPTIMIZATION**

Sherwood H. Tiffany and William M. Adams, Jr.

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**FITTING AERODYNAMIC FORCES IN THE LAPLACE DOMAIN:
An Application of a Nonlinear Nongradient Technique To
Multilevel Constrained Optimization**

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SUMMARY

A technique which employs both linear and nonlinear methods in a multilevel optimization structure to best approximate generalized unsteady aerodynamic forces for arbitrary motion is described. Optimum selection of free parameters is made in a rational function approximation of the aerodynamic forces in the Laplace domain such that a best fit is obtained, in a least squares sense, to tabular data for purely oscillatory motion. The multilevel structure and the corresponding formulation of the objective models are presented which separate the reduction of the fit error into linear and nonlinear problems, thus enabling the use of linear methods where practical. Certain equality and inequality constraints that may be imposed are identified; a brief description of the nongradient, nonlinear optimizer which is used is given; and results which illustrate application of the method are presented.

INTRODUCTION

This paper describes a method of determining a constrained approximation to the generalized aerodynamic forces for arbitrary motion. Nonlinear rational functions, as described in references 1-5, are used to fit tabular aerodynamic force data for purely oscillatory motion in a least squares sense with certain specified constraints enforced. An error function which represents the weighted deviations between the approximating functions and the tabulated data at specified values of frequencies is minimized by separating the error function into two objective functions. Linear and nonlinear techniques are then employed independently in the optimization process. The objective function for the linear portion of the problem includes the equality constraints which can be imposed on the approximating functions (Refs. 3,4). This "linear" portion of the problem can be solved algebraically, using matrix techniques. The "nonlinear" objective function is reduced by employing a simple sequential simplex (or polytope) algorithm (Refs. 6-8) for nonlinear unconstrained minimization. This particular optimizer is employed because it is a nongradient, numerically stable algorithm which has been found to converge rapidly in numerical applications. Side constraints are imposed on the design variables to ensure system stability.

Once the approximating functions are determined, the rational formulation allows transformation of the equations of motion into linear time invariant (LTI) form. Optimum selection of the design variables offers the potential for a good fit to the tabulated aerodynamic data with fewer design variables which translates into a smaller LTI set of system equations. This possibility is highly desirable and provided the impetus for the research described herein.

The method of separating the problem objectives in such a way that the optimization can be performed using linear and nonlinear methods independently was employed in reference 9. Furthermore, the form of the approximating functions can be shown to

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be equivalent to the matrix-Padé approximation as implemented therein. Basic differences do exist, however, in that various equality constraints on the aerodynamic fits are included here, and a nongradient algorithm is employed to perform the nonlinear optimization.

SYMBOLS

A	Matrix multiplier of states used in the LTI representation of the equations of motion, $\dot{X} = AX + BU$.
A_0, A_1, \dots	Coefficients used in rational function approximation (RFA) for an element of matrix Q . See equation (7).
\bar{A}_{ij}	Column vector of coefficients, $(A_0, A_1, \dots)^T$, for the (i,j) element of matrix \hat{Q} . See equations (8) and (11).
b_ℓ	Constant term in the ℓ th denominator of a rational function approximation, Q , for an element of matrix Q . This term in the RFA is commonly referred to as a "lag" term. See equation (7).
\bar{b}_j	Vector of denominator constants used in the lag terms of the RFA's for the j th column of \hat{Q} . See equation (15).
B	Matrix multiplier of inputs in the LTI representation of equations of motion, $\dot{X} = AX + BU$.
$L_{\ell j}, U_{\ell j}$	Lower and upper bounds on ℓ th lag coefficient for the j th column of \hat{Q} . See equation (16).
C_j	Matrix multiplier of unknown coefficients used in defining linear equality constraints for the j th column of \hat{Q} . See equations (11) and (12).
C_0, C_{0ij}	Matrix of constants used in defining linear equality constraints for the j th column of Q . See equations (11) and (12).
$[D]$	Matrix of damping coefficients used in Lagrangian Formulation of the equations of motion. See equations (2) and (3).
$E_j(\bar{b}_j)$	The total error in the approximations over the j th column of the matrix Q . Specifically, it is the weighted sum of square errors in the approximations to the elements in the j th column of the Q matrix as a function of the lag coefficients currently being used for that column. See equation (15).
$\{f(t)\}$	Vector of forcing functions in the time domain. See equation (2).
$\{F(s)\}$	Vector of forcing functions in the Laplace domain. See equations (3) and (4).
i	Index over which a summation is performed, see equation (1), or the complex variable $\sqrt{-1}$. As a subscript or superscript, it refers to the i th row of a matrix.

j	As a subscript or superscript it is used to refer to the j th column of a matrix.
$[K]$	Generalized stiffness matrix used in the Lagrangian formulation of the equations of motion. See equations (2) and (3).
ℓ	As a subscript, it refers to one of the "lag" terms in the rational function approximations (see Eq. (7)). As a subscript in a partial derivative of equations (7), (10) and (13), it refers to one of the A-coefficients or λ -coefficients.
$[M]$	Generalized mass matrix used in the Lagrangian formulation of the equations of motion. See equations (2) and (3).
n_ℓ	Number of lag terms used in a specific approximation. See equation (8).
n_q	Number of modes retained in the EOM to define the motion of the vehicle.
ΔP	Lifting pressure at a specified point on the upper surface of the aircraft equal to the difference in pressures between the upper and lower surface. See equation (5).
ΔP_j	The change in the lifting pressure due to changes in the j th generalized coordinate. See equation (5).
$q_i(t)$	Generalized coordinate representing the amplitude of the corresponding i th component mode shape in the total deflection as a function of time. See equation (1).
$\{q\}, \{\dot{q}\}, \{\ddot{q}\}$	The vector of generalized coordinates and its first two time derivatives used in the Lagrangian formulation of the equations of motion. See equation (2).
$\{\tilde{q}(s)\}$	Vector of generalized coordinates in the Laplace domain. See equations (3) and (4).
$Q(s)$	An element of $Q(s)$ as in equation (9).
$Q(s)$	Matrix of generalized force coefficients. See equation (4).
$\hat{Q}(s)$	The rational function approximation to an element of $Q(s)$. See equations (7) and (14).
$\hat{Q}(s)$	The matrix of rational function approximations of $Q(s)$.
s	Laplace complex variable.
S	Total lifting surface of the aircraft. See equation (5).
t	Time variable.

w_{ij}	The weight applied to the i th row fit error in the j th column error, E_j . See equation (15).
x	The x -coordinate in an (x,y,z) spacial coordinate system.
X, \dot{X}	The vector of states and its derivative, respectively, in the LTI formulation of the equations of motion.
y	The y -coordinate in an (x,y,z) spacial coordinate system.
$z, z(x,y,t)$	The z -coordinate in an (x,y,z) spacial coordinate system representing the deflection at a point (x,y) at a given time, t .
$z_1, z_1(x,y)$	The time independent mode shape component of the deflection z . See equation (1).
z_ℓ	The design variable in the entire real space corresponding to $b_{\ell j}$ in the sinusoidal transformation given by equation (17).
ϵ	The unconstrained least square error in the fit of an element of Q as given by equation (9).
ϵ_{ij}^c	The objective function in the Lagrangian formulation of the error in the constrained fit of the (i,j) element of Q as defined by equation (12).
λ	A Lagrangian multiplier used to include an equality constraint in the optimization of an objective function.
$\bar{\lambda}_{ij}$	The vector of Lagrangian multipliers in the Lagrangian formulation of the constrained objective function for the (i,j) element of Q as given by equation (12).
ω	Frequency of oscillation.
ω_k	A specified value of frequency at which tabular values for the generalized aerodynamic forces are determined.
$[]^T$	The transpose of a matrix $[]$.

PROBLEM DEFINITION

Equations of Motion

Some motions of a flexible aircraft are a result of the induced downwash of the airflow over a surface due to motion of the airfoils themselves. These unsteady aerodynamic effects as well as the structural dynamics must be considered when modeling an actively controlled flexible aircraft. Using a Lagrangian approach and considering only small perturbations from a level equilibrium flight condition, the perturbed aircraft is represented by a linearized system of equations expressed in terms of generalized coordinates (Ref. 10). As depicted in figure 1, the structural displacement, $z(x,y,t)$ can be represented by the sum of products of a finite set of

both shape functions (mode shapes) and time functions through the method of separation of variables:

$$\text{Displacements: } z(x,y,t) = \sum_1 [z_1(x,y,0) q_1(t)] \quad (1)$$

The equations of motion (EOM) can be represented in the time domain as

$$\text{EOM (time domain): } [M]\{\ddot{q}\} + [D]\{\dot{q}\} + [K]\{q\} = \{f(t)\} \quad (2)$$

where the generalized mass and stiffness matrices are integral functions of the mode shapes. The entire system of equations may then be transformed into the Laplace domain where algebraic methods can be more readily employed to examine system stability.

$$\text{EOM (Laplace domain): } ([M]s^2 + [D]s + [K])\{\tilde{q}(s)\} = \{F(s)\} \quad (3)$$

The forces on the right-hand side of equations (2) and (3) are a result of the aerodynamic forces due to aircraft motion, turbulence and control surface motions. In the Laplace domain, the unsteady aerodynamic forces can be expressed in the form

$$\{F(s)\} = [Q(s)]\{\tilde{q}(s)\} \quad (4)$$

where the matrix Q is called the matrix of aerodynamic force coefficients. It is this matrix of coefficients which is of interest here.

Generalized Aerodynamic Forces

Figure 2 depicts the fact that these aerodynamic force coefficients may be obtained as a surface integral function of mode shapes and the changes in lifting pressure due to downwash in each of the modes:

$$Q_{1j}(s) = \iint_S z_1(x,y) \Delta P_j(x,y,s) dS \quad (5)$$

$$\Delta P_j: \partial(\Delta P)/\partial \tilde{q}_j$$

s : Laplace variable

S : Total surface

The programs currently available for production generation of aerodynamic forces can only compute these forces for purely oscillatory motion

$$s = i\omega$$

for specified values of frequency. Hence, each element of $Q(s)$ is defined only for a finite set of values. An unsteady aerodynamic lifting surface theory commonly referred to as the Doublet Lattice method, described in references 11 and 12, was used to obtain the tabular values for the generalized aerodynamic force coefficients herein.

The fact that the generalized aerodynamic force coefficients can only be obtained for purely oscillatory motion is one of the major problems in solving the equations of motion for stability characteristics, etc. (Ref. 13). Since the aerodynamic forces are transcendental functions of ω and are available only in tabular form for a finite set of frequencies, iterative solution methods (which tend to be costly) must be used to determine system stability. Furthermore, the solutions are only exact for purely oscillatory motion, $i\omega$.

Aerodynamic Force Coefficients for Arbitrary Motion

In order to obtain solutions in the Laplace domain for both growing and decaying motion (s off the $i\omega$ axis), it is necessary to express the forces as a function of s . The concept of analytic continuation is often used to circumvent the difficulty of having the generalized aerodynamic force coefficients only as tabular values of $s = i\omega$. Furthermore, it is desirable that the functional relationship be rational to enable writing the EOM in linear time-invariant (LTI) state space form:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{U} \quad (6)$$

Hence, efficient linear system techniques can be employed to solve for system stability, etc. (Refs. 2-5).

Rational Function Approximations

The form of the approximation is generally written as

$$\hat{Q}(s) = A_0 + A_1 s + A_2 s^2 + \sum_{\ell=1}^{n_\ell} A_{(\ell+2)} \frac{s}{s + b_\ell} \quad (7)$$

for each element of $Q(s)$. The problem then becomes one of fitting the available tabular data as well as possible at each specified value of frequency, ω_k . The fact that the fit is not specified at all values of frequency means that the rational function is only an approximate analytic continuation. Accurateness of the fit depends not only on the number of frequencies for which tabular data are available, but also upon the number, n_ℓ , of "lag" coefficients used in the approximation. However, as figure 3 depicts, even when the same set of b_ℓ is used for a given column of Q , the number of aerodynamic states introduced into the LTI formulation of the EOM is equal to the number of modes retained to define the motion times the total

number of "lag" terms employed in the approximations. Therefore, in order to keep the order of the matrix equations down to a reasonable size, computationally, it is desirable to reduce the number of b_l employed in the fits to be as few as possible without adversely affecting the overall system analyses. For this reason, the number of these lag coefficients used in the approximations is a critical factor in the analyses for which these approximations are being determined. The primary reason for the multilevel, nonlinear optimization structure being demonstrated is to keep n_l as small as possible.

MULTILEVEL OPTIMIZATION STRUCTURE

Unconstrained Linear Optimization

The rational approximation is linear with respect to the coefficients, A_0 , A_1 , etc. If the coefficients,

$$\bar{A} = (A_0, \dots, A_{n_l+2})^T \quad (8)$$

are used as design variables for a particular element of $Q(s)$, and if a square error function

$$\epsilon = \sum_k |Q(i\omega_k) - \hat{Q}(i\omega_k)|^2 \quad (9)$$

satisfies the minimum condition that all partials with respect to each design variable be 0

$$\frac{\partial \epsilon}{\partial A_l} = 0, \quad (10)$$

the resulting system of equations is linear, and the algebraic solution for the coefficients giving the least square error is possible. Henceforth, this is referred to as a "linear" optimization even though the objective function is quadratic since linear, algebraic methods may be employed to determine the optimum solution. Figure 4 shows the least square error fit to a single element in the Q matrix which resulted when two "lag" terms were employed:

$$\frac{s}{s + b_1} \quad \& \quad \frac{s}{s + b_2}$$

The circles are the actual tabular values, and the solid dots are the corresponding values of the approximation at the same values of frequency. It should be noted that the fit at zero frequency, which represents the steady state conditions, is extremely poor. Since steady state characteristics of the system are often known, it might be desirable to be able to impose certain equality constraints on the system, such as equality between the approximations and the tabular data for $\omega = 0$ (Refs. 3,4).

Linear Equality Constraints

There are actually several types of equality constraints which might be desirable to impose on the rational approximations. In addition to forcing agreement at $\omega = 0$, one may wish either to impose some constraint on the slope at $\omega = 0$ or to null out a specific coefficient, such as A_2 , in order to eliminate acceleration terms for a specific element in the force matrix. Furthermore, since flutter (the point at which the vehicle becomes unstable) is a critical element in aircraft flight analysis, it might also be desirable to force agreement near a known flutter frequency.

The linear equality constraints which have been implemented (refer to appendix A in Ref. 5) are to:

- a) Constrain approximating functions at low frequency ($\omega = 0$) to be the actual tabular values $Q_{ij}(0)$.
- b) Constrain slopes of approximating functions at $\omega = 0$ to be specified values.
- c) Null out specific coefficients in the approximations.
- d) Force agreement of approximating functions with interpolated values at some specified frequency, such as the flutter frequency.

Each constraint is imposed on an entire column of $Q(s)$, and can be expressed in terms of linear equations involving the design variables, A_{ij} , as

$$C_j \bar{A}_{ij} = C_{0_{ij}} \quad (11)$$

Again, refer to reference 5, Appendix A, for details.

Constrained Linear Optimization

The Lagrange multiplier formulation for including equality constraints is employed to redefine the fit error for a given (i,j) element of the aerodynamic force coefficient matrix,

$$\epsilon_{ij}^c = \sum_k \left| Q_{ij}(i\omega) - \hat{Q}_{ij}(i\omega) \right|^2 + 2\bar{\lambda}_{ij}^T (C_j^T \bar{A}_{ij} - C_{0_{ij}}) \quad (12)$$

The corresponding gradient conditions

$$\frac{\partial \epsilon_{ij}}{\partial A_{\ell ij}} = 0 \quad \text{and} \quad \frac{\partial \epsilon_{ij}^c}{\partial \lambda_{\ell ij}} = 0 \quad (13)$$

define a linear system of equations whose algebraic solution is a least square error approximation which exactly satisfies the imposed constraints.

Figure 5 shows a constrained approximation to the same element of Q as shown in figure 4 for which the function and its slope are specified at 0 frequency. As the figure indicates, the imposed constraints at zero frequency have been satisfied, but the fit at higher frequencies has deteriorated drastically. However, there are additional free parameters in the rational function approximation, namely the b_{ℓ} , which can be optimized to improve the fits. Up to this point in the approximations, these have been specified a priori over the range of desired frequencies based upon engineering judgement. Historically, these parameters, or their equivalent in the matrix-Padé approximants, have not been included as design variables since the resulting gradient equations, unlike equations (13), would not be linear. The technique employed herein to optimize these is a nongradient-based method. Reference 9 defines a method of optimizing the equivalent parameters in the matrix-Padé approximants using gradient methods.

Linear versus Nonlinear

Figure 6 depicts the separation of the design variables for the "linear" and "nonlinear" optimization. The overall optimization problem can then be defined by considering as design variables

$$\bar{b}_j = (b_{1j}, \dots, b_{n_{\ell}j}) \quad (14)$$

for a given column and all the linear coefficients, \bar{A}_{ij} , for each element of the same column. The optimization can then be performed as a sequence of linear problems over the specified column for a fixed set, \bar{b}_j , of $b_{\ell j}$'s which are sequentially selected by a nonlinear optimizer.

Objective Functions

The actual optimization of \bar{b}_j is achieved by using equation 12, the constrained fit error for a specific element of the Q-matrix, as a "linear" objective function per element. A weighted sum of these square element errors over the specified column is used as the "nonlinear" objective function:

$$E_j(\bar{b}_j) = \sum_1 w_{ij} \epsilon_{ij}^c \quad (15)$$

The weights might be defined to improve the approximations for some elements of Q (larger w_{ij}) while decreasing the importance of others (smaller w_{ij}).

Design Constraints for Nonlinear Optimization

Since \bar{b}_j is now allowed to vary, it is necessary to impose side constraints on its elements. The b_{lj} must be greater than 0 in order to ensure system stability as a result of introducing the related aerodynamic states into the state equations. Also, it is frequently desired to restrict the variation to that range of frequencies over which tabular data is available.

$$(0 \leq) L_{lj} < b_{lj} < U_{lj} \quad (16)$$

These side constraints may be enforced in one of two ways. The first is by way of a sinusoidal transformation of the real line segment $[-1, +1]$ into the design space defined by (16) (see Ref. 14).

$$b_{lj} = (U_{lj} - L_{lj})/2 \sin(\pi/2 z_l) + (L_{lj} + U_{lj})/2, \quad -1 < z_l < 1 \quad (17)$$

This transformation ensures that the side constraints are always satisfied. However, since the restrictions on the z_l are not strictly enforced, on occasion an oscillation problem between successive values of the design parameters arises due to the multi-valued characteristic of this transformation. Various methods could be applied to avoid this problem. One could modify the original nonlinear optimization code. However, since the problem rarely occurs, a penalty function formulation of the nonlinear objective function is usually employed (Refs. 15,16) when it does. Two examples of available penalty functions are a "wall" function and the extended-interior penalty function. By "wall," we mean that the function essentially hits a wall when constraints are violated. This technique is implemented by defining the objective function as very large relative to its normal range of values. This obviously has extreme discontinuities which can sometimes cause convergence problems and usually requires that an optimization process start within the feasible design space in which no constraints are violated. The second type of penalty function which may be selected by the engineer is an extended-interior formulation proposed by Haftka and Starnes (Ref. 15). It places no initial restrictions and does not inject discontinuities into the convergence process. This is the preferred type of penalty function since convergence in this case is smoother.

Overall Program Flow

Referring to figure 7, the overall optimization can be summarized as follows. The engineer first determines the form of the RFA to be used and the columns over which the optimization is to be performed. The process is then started by selecting an initial set of $\{b_{lj}\}$ and design weights, $\{w_{ij}\}$. The vectors of coefficients \bar{A}_{ij} which minimize the "linear" equality-constrained objective functions, ϵ_{ij}^c are then determined for each element in the desired column of the Q -matrix. The nonlinear objective function, $E_j(\bar{b}_j)$, is evaluated and a new set of values for the

design variables, $\{b_{kj}\}$, is determined by the nonlinear optimizer. The best set of A_{ij} 's is then redetermined using linear methods. This process continues until either a reasonable minimum error is achieved or the engineer decides no further progress can be made with the current approximations. Even when a minimum error is achieved, the fits may not be satisfactory. At this point, the engineer must intervene and select a different set of initial lag coefficients, increase the number of lag coefficients, change the linear constraints, etc., until the approximations are satisfactory.

NONGRADIENT NONLINEAR OPTIMIZER

Sequential Simplex

The nonlinear optimizer used in this multilevel structure is a sequential simplex (or polytope) method originally proposed by Nelder and Mead (Refs. 6-8). It has been used in control system design for flexible aircraft with active controls (Refs. 17,18), along with gradient methods such as the CONMIN - feasible direction method (Ref. 19), the Davidon-Fletcher-Powell variable metric (Ref. 20,21), and the newer Davidon Optimally Conditioned method (Ref. 22). It has been found that this method is extremely simple to use, requiring little or no "fine-tuning." It is an adaptive method which moves away from "high" points. It requires no gradients and is a numerically stable and robust algorithm. Since computation of gradients using finite differencing is costly in large application programs for which closed-form gradients are either not available or not practical, this nongradient algorithm has proven to be invaluable. It lacks the initial sensitivities of gradient-based methods. But, the final two low values and the stepsize between them do provide some measure (although somewhat obscure) of sensitivity at convergence. At this point, the gradient could be computed if desired.

The basic mathematical justification of convergence for this algorithm is based only on the convergence of a monotonically decreasing series. Although its order of convergence depends upon the function being optimized, it has been found to converge fairly rapidly in numerical applications.

Description of Algorithm

Figure 8 depicts the algorithm for a two-dimensional design space. The algorithm starts with a simplex of points in the design space ($\triangle ABC$). The objective function is evaluated at each vertex in the simplex and the highest-valued point is identified (A). A line of projection through the centroid of the "opposite side" (namely, the other n vertices) is determined and the objective function is then evaluated at the reflected point (E) on the projection line. Depending upon the relative value of the function at this point and the other points in the simplex, a single extension (F), a retention (E), an exterior contraction (G), or an interior contraction point (H) is then identified at which the objective function is evaluated and a new simplex is determined. The actual decision process is detailed in references 6-8 although the code as listed (Ref. 7) deviates from the decision process as described in the reference with respect to those steps taken when equalities hold.

Actual Fits

Figure 9 shows the plots of the actual RFA approximations to the original tabular data for one element of the Q -matrix. The circles represent the tabular data, and the solid dots represent the values of the approximation at the corresponding frequencies. In both cases, the improvement as a result of including the b_l 's in the optimization process is significant. Although the "2-lag" case gives a questionable fit, even after optimization of the lag coefficients, the "optimized fit" is an approximation which might be satisfactory for an initial design of a control system. The obviously more acceptable fit of the "4-lag" case would be preferable for final analyses. The "4-lag" case, however, introduced an additional 24 states. This resulted in a total of 48 aerodynamic states due only to the lag coefficients in the state-space equations for which these approximations were being used.

CONCLUSIONS

In conclusion, the multilevel structure of the optimization is a natural result of the form of the approximating functions and provides obvious benefits. It reduces cost since iterative solutions are only required for the nonlinear portion of the design problem. Closed form algebraic solutions of the "linear" optimization portion of the problem do not rely on the convergence properties of the "nonlinear" portion.

The nongradient Nelder-Mead sequential simplex algorithm is reliable and efficient. The method is simple to use and robust in its ability to handle the nonlinear optimization problem to which it has been applied. Its adaptive nature requires minimal effort on the part of the engineer to "fine-tune" the problem. It has provided a cost effective means by which to include "lag" coefficients in the optimization of the rational function approximations to the generalized aerodynamic forces in the Laplace domain. Optimization of the lag coefficients can reduce the number of lag coefficients required to achieve good fits to the tabular unsteady aerodynamic force data while enforcing needed constraints upon those fits. This enables the use of smaller linear time-invariant systems in the analysis and design of flight control systems for aeroelastic aircraft.

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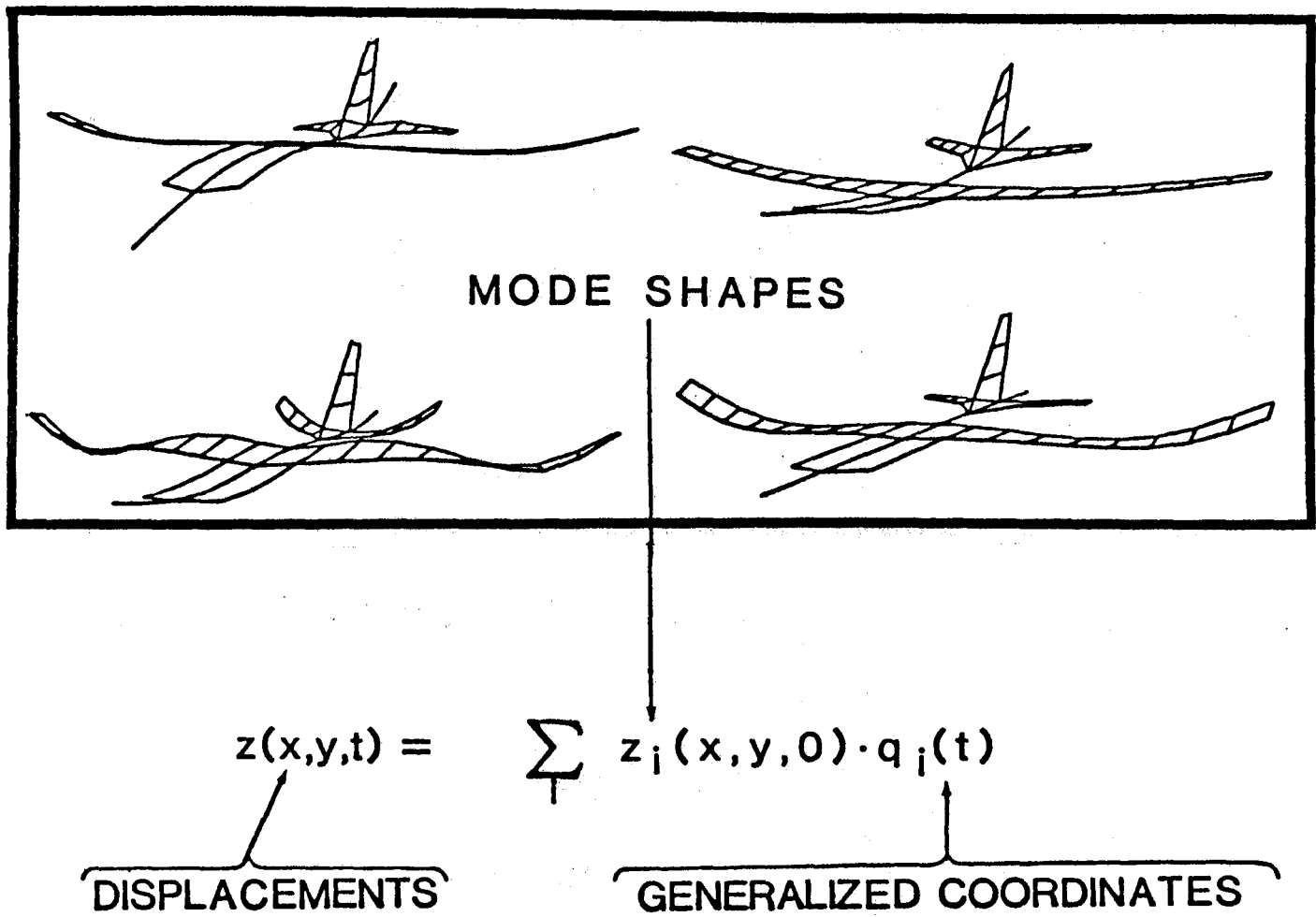
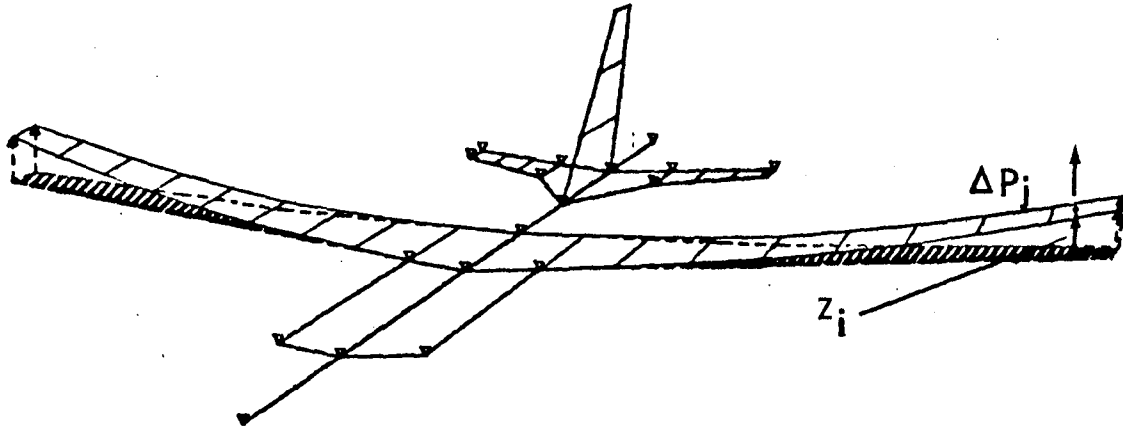


Figure 1.- Displacements in terms of modeshapes and generalized coordinates



$$Q_{ij}(s) = \iint_S z_i(x,y) \Delta P_j(x,y,s) dS$$

ΔP_j : $\partial(\Delta P)/\partial \tilde{q}_j$

s : Laplace variable

S : surface

\tilde{q}_j : Laplace transform of j th
generalized coordinate

Figure 2.- Generalized Aerodynamic Forces

STATE SPACE EOM:

$$\dot{X} = AX + BU$$

$$\left[\begin{array}{c} \downarrow \\ Q_{ij} \end{array} \right]$$

SAME b_ℓ FOR GIVEN COLUMN

AERO. STATES = (# MODES) \times (# LAG COEFF.)

$$n_q \times n_\ell$$

⇒ REDUCE NUMBER OF b_ℓ

Figure 3.- Aerodynamic states.

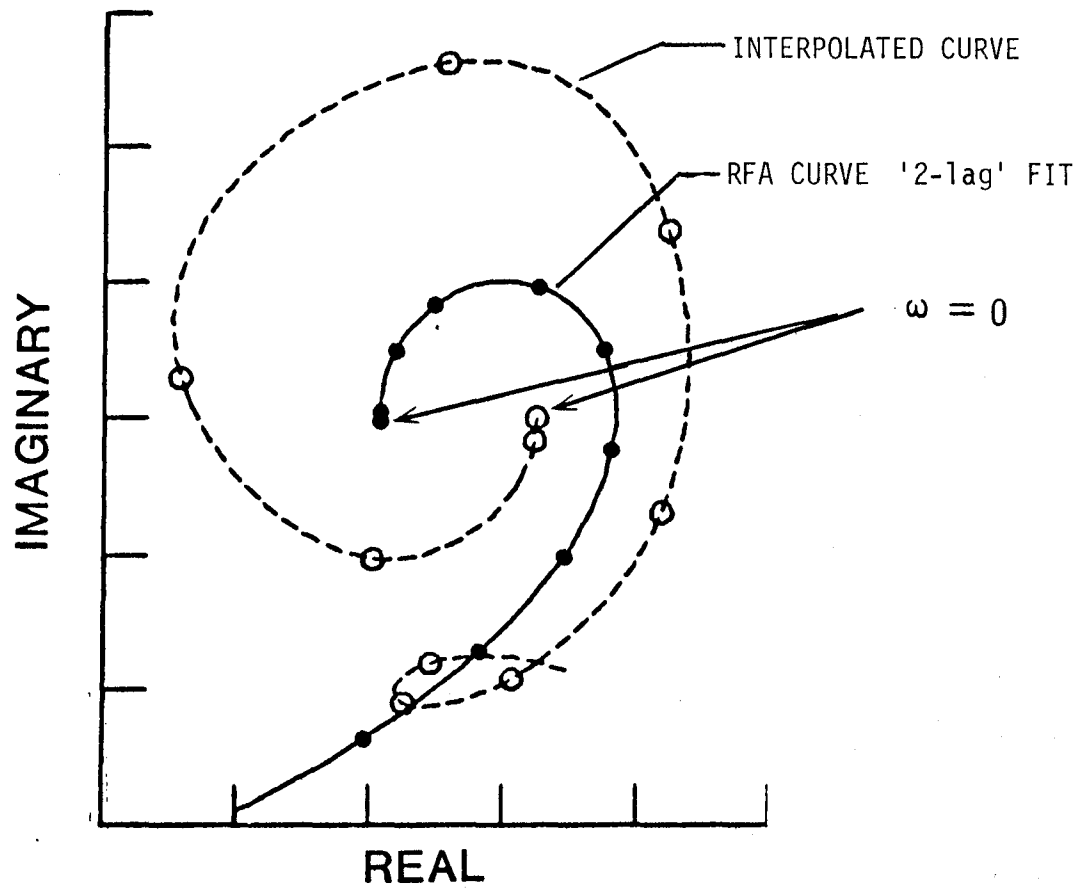


Figure 4.- Initial unconstrained '2-lag' fit (unoptimized \bar{b}_j).

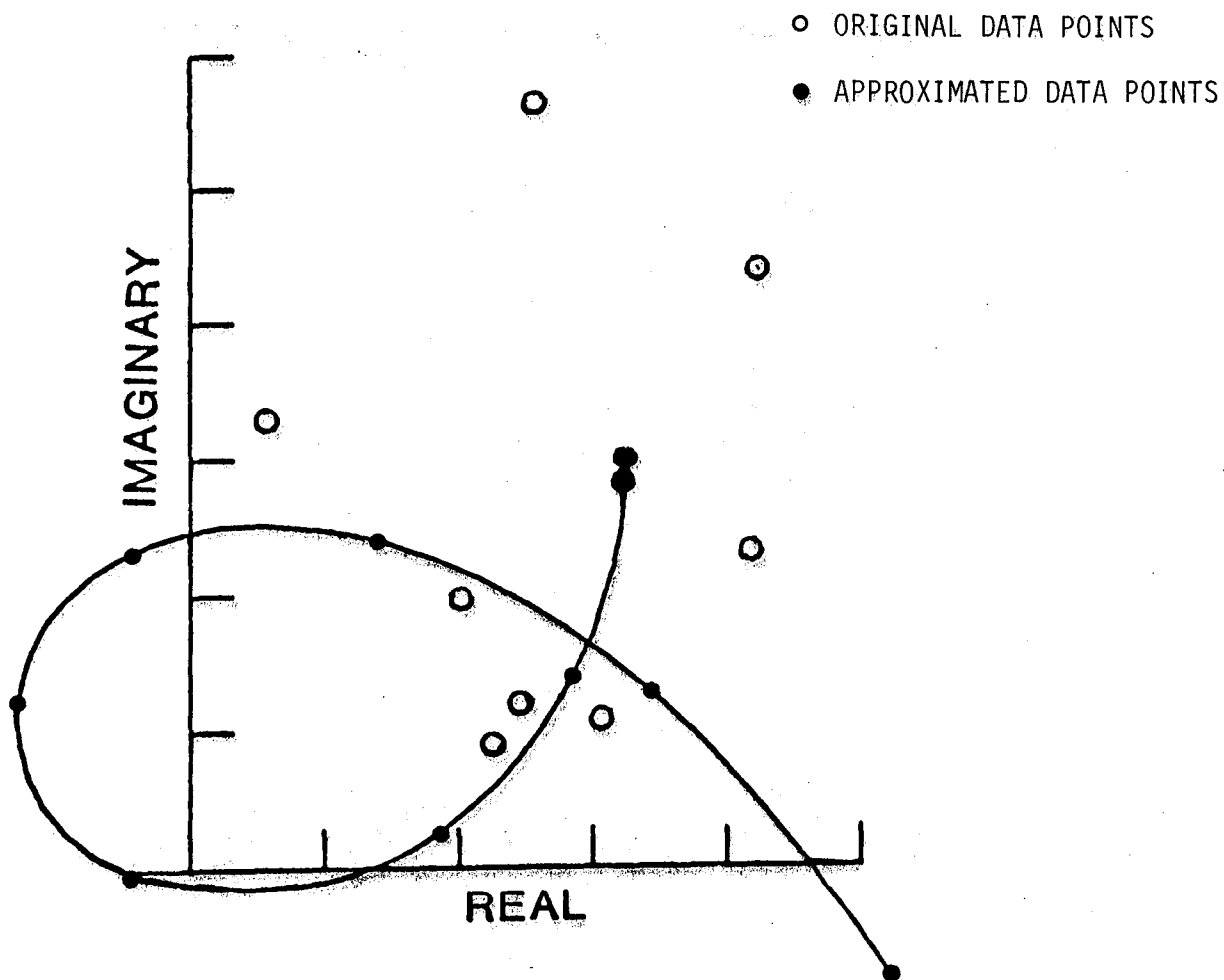


Figure 5.- Initial constrained '2-lag' fit (unoptimized \bar{b}_j).

$$\hat{Q}(s) = \boxed{A_0} + \boxed{A_1}s + \boxed{A_2}s^2 + \sum_{\ell=1}^{n_\ell} \boxed{A_{(\ell+2)}} \frac{s}{s + \boxed{b_\ell}}$$

DESIGN VARIABLES: $d = (\bar{b}_j, \boxed{\bar{A}_{1j}}, \dots, \boxed{\bar{A}_{n_{qj}}})$

$$\bar{b}_j^T = (\dots, b_\ell, \dots)_j$$

$$\boxed{\bar{A}_{1j}^T} = (A_0, A_1, \dots, A_{n_\ell+2})_{1j}$$

(a) 'Linear' Optimization

$$\hat{Q}(s) = A_0 + A_1s + A_2s^2 + \sum_{\ell=1}^{n_\ell} A_{(\ell+2)} \frac{s}{s + \boxed{b_\ell}}$$

DESIGN VARIABLES: $d = (\boxed{b_j}, \bar{A}_{1j}, \dots, \bar{A}_{n_{qj}})$

$$\boxed{\bar{b}_j^T} = (\dots, b_\ell, \dots)_j$$

$$\bar{A}_{1j}^T = (A_0, A_1, \dots, A_{n_\ell+2})_{1j}$$

(b) 'Nonlinear' Optimization

Figure 6.- Multilevel optimization structure.

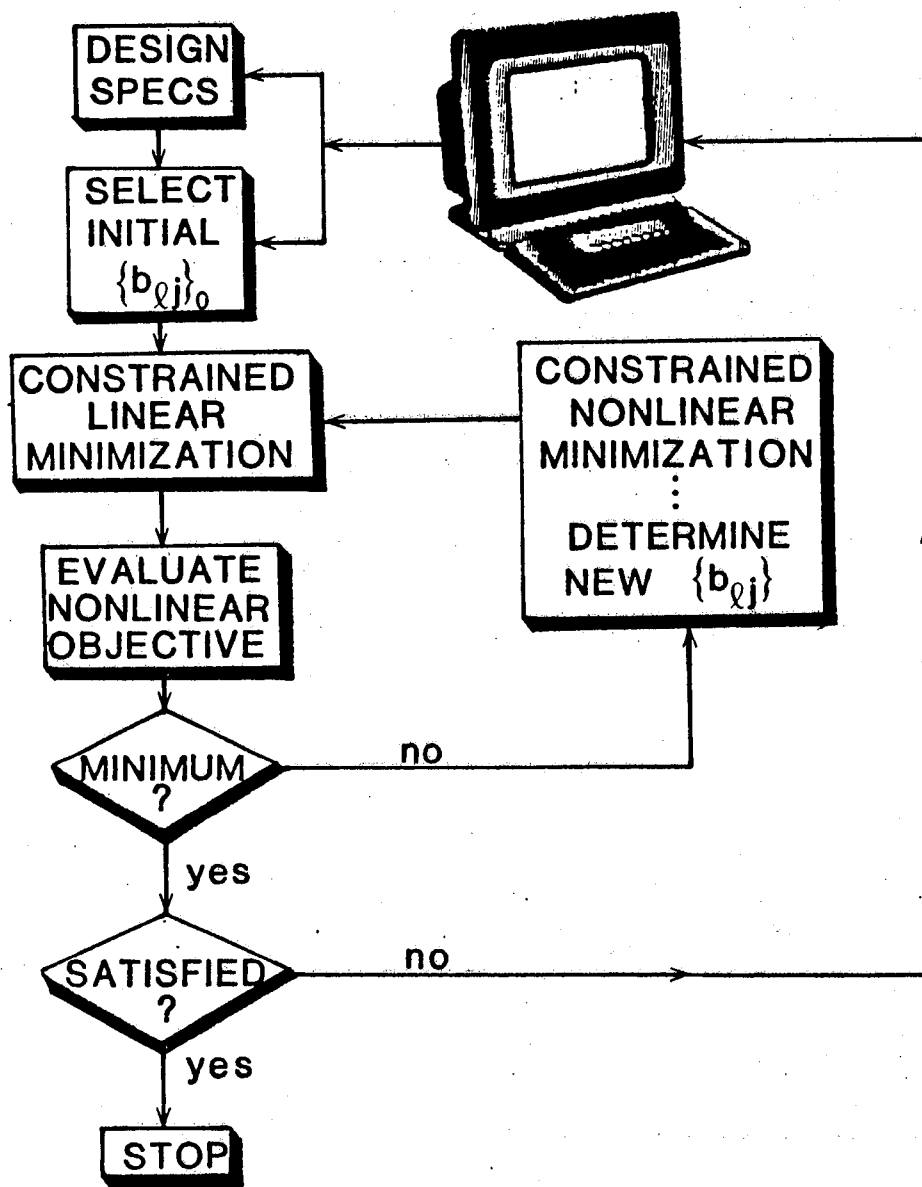
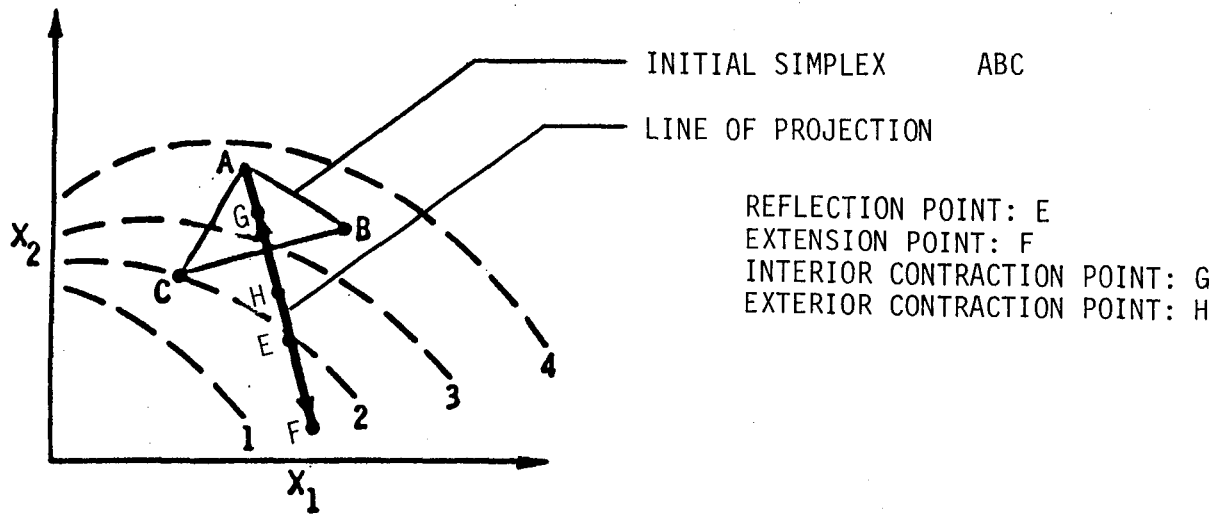
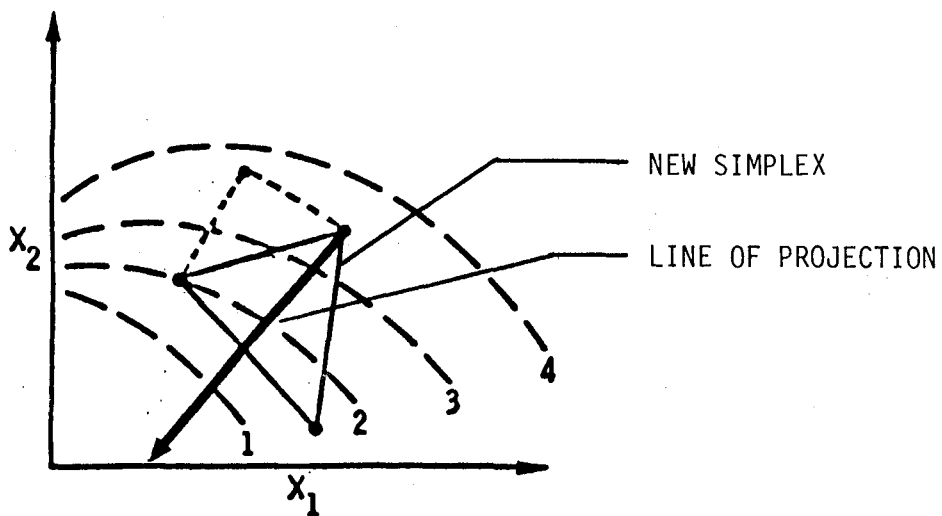


Figure 7.- Multilevel optimization flow.

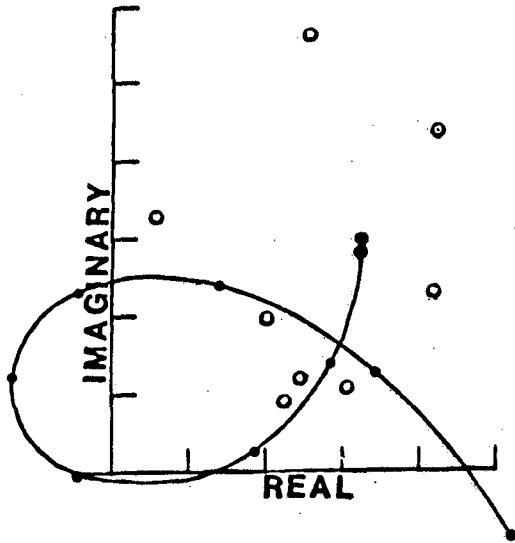


(a) Initial Simplex and Line of Projection

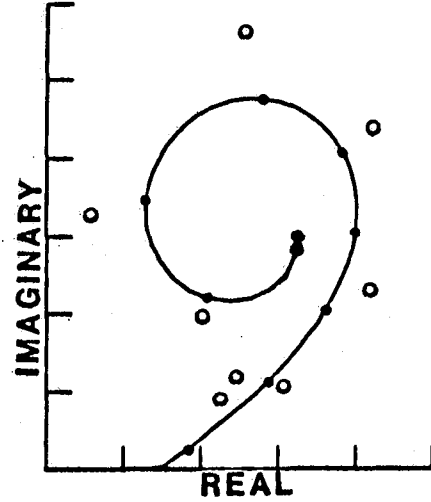


(b) New Simplex and Line of Projection

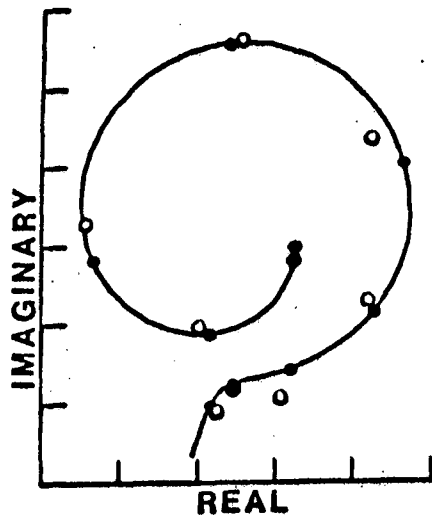
Figure 8.- Two dimensional view of adaptive sequential simplex algorithm.



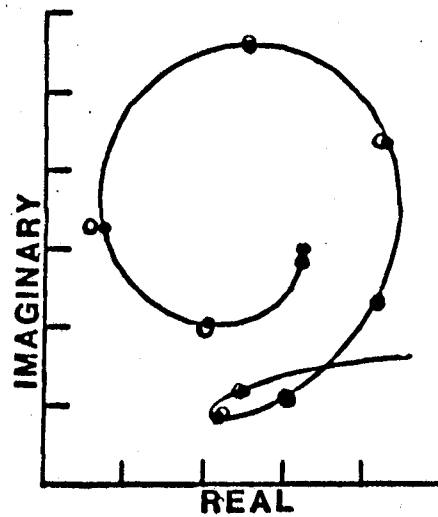
(a) '2-lag' fit, unoptimized \bar{b}_j



(b) '2-lag' fit, optimized \bar{b}_j



(c) '4-lag' fit, unoptimized \bar{b}_j



(d) '4-lag' fit, optimized \bar{b}_j

Figure 9.- Constrained Rational Function Approximations to original tabular data for a single element of the Q-matrix.

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16. Abstract <p>A technique which employs both linear and nonlinear methods in a multilevel optimization structure to best approximate generalized unsteady aerodynamic forces for arbitrary motion is described. Optimum selection of free parameters is made in a rational function approximation of the aerodynamic forces in the Laplace domain such that a best fit is obtained, in a least squares sense, to tabular data for purely oscillatory motion. The multilevel structure and the corresponding formulation of the objective models are presented which separate the reduction of the fit error into linear and nonlinear problems, thus enabling the use of linear methods where practical. Certain equality and inequality constraints that may be imposed are identified; a brief description of the nongradient, nonlinear optimizer which is used is given; and results which illustrate application of the method are presented.</p>					
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